Coupled-mode theory of the Raman effect in silicon-on-insulator waveguides

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Coupled-mode theory is used to calculate Raman gain and spontaneous efficiency in silicon waveguides with cross-sectional areas ranging from 0.16 to 16 μm^2. We find a monotonic increase in the Raman gain as the waveguide cross section decreases for the range of dimensions considered. It is also found that mode coupling between the Stokes modes is insignificant, and thus polarization multiplexing is possible. The results also demonstrate that for submicrometer waveguide dimensions the Einstein relation between spontaneous efficiency and stimulated gain no longer holds. © 2003 Optical Society of America

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Raman amplification in silicon-on-insulator (SOI) waveguides was recently considered as a potential approach to developing active silicon photonic circuits.[1,2] Raman gain has been demonstrated on III–V compound semiconductor (GaP) waveguides.[3,4] The Raman process has a higher significance in silicon because of (i) the technological importance of silicon and (ii) the lack of other practical means to achieve light amplification and emission. Previously, we estimated that a gain as high as 10 dB may be achieved in 2-cm SOI waveguides with cross sections of the order of 2 μm^2, with pump powers of the order of ~200–500 mW.[1] In this Letter we calculate the Raman gain in SOI waveguides with order-of-magnitude smaller areas. At such dimensions the interaction between the pump and the signal is a strong function of the energy distribution within the waveguide. Hence, to calculate the gain accurately, one must invoke the coupled-mode formulation. As we will show, for submicrometer waveguide dimensions, the Einstein relation between spontaneous efficiency and stimulated gain no longer holds. We also find that coupling between TE and TM Stokes modes is negligible. This negligible coupling, combined with a nearly isotropic gain coefficient in the chosen waveguide orientation, permits polarization multiplexing of two signal channels.

The Raman effect in silicon is due to the scattering of light by the optical phonons of the crystal. The strongest Stokes peak (first order) is due to scattering from the threefold degenerate optical modes at the center of the Brillouin zone.[5] The induced polarization, $\vec{\mathbf{P}}_i$, responsible for the Stokes radiation associated with the ith component of phonon displacement is[6]

$$\vec{\mathbf{P}}_1(\omega_s) = \epsilon_0 \chi_R \vec{\mathbf{R}}_1 \cdot \vec{\mathbf{E}}(\omega_p)\,,$$
$$\vec{\mathbf{P}}_2(\omega_s) = \epsilon_0 \chi_R \vec{\mathbf{R}}_2 \cdot \vec{\mathbf{E}}(\omega_p)\,,$$
$$\vec{\mathbf{P}}_3(\omega_s) = \epsilon_0 \chi_R \vec{\mathbf{R}}_3 \cdot \vec{\mathbf{E}}(\omega_p)\,, \qquad (1)$$

where χ_R is the scalar susceptibility and the Raman tensors $\vec{\mathbf{R}}_i$ determine the polarization of the Stokes wave. We are mainly interested in scattering in waveguides fabricated parallel to the $[1\bar{1}0]$ direction on a silicon [001] surface. This interest is due to the favorable cleaving property of silicon in this orientation. We define the coordinate system (x, y, z) that is rotated with respect to the crystallographic-axes system by 45° rotation around the [001] axis. The Raman tensors in the chosen coordinate system have the form

$$\vec{\mathbf{R}}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \qquad \vec{\mathbf{R}}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix},$$
$$\vec{\mathbf{R}}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \qquad (2)$$

The scattering efficiency, which is the percentage of scattered radiation per unit of solid angle per unit length of material, is given by

$$S = S_0 \sum_{n=1,2,3} |\hat{e}_s \cdot \vec{\mathbf{R}}_n \cdot \hat{e}_i|^2, \qquad S_0 = \frac{{k_0}^4}{32\pi^2 n} V {\chi_R}^2, \quad (3)$$

where $\hat{e}_i$ and $\hat{e}_s$ are the polarizations of the incident and the scattered radiation, respectively, k_0 is the Stokes wave vector, and n is the index of refraction. The value of the scattering efficiency for a Stokes wavelength of 1.55 μm can be calculated from the published value at the 1.06-μm pump wavelength,[7] since the Raman susceptibility is nondispersive in this frequency range.[8] By accounting for the λ^{-4} dependence, we find that $S_0 = 8.4 \times 10^{-7}\ \mathrm{cm^{-1}\,Sr^{-1}}$.

For the stimulated Raman effect in bulk material, the gain coefficient, g_R, is obtained from the spontaneous efficiency from the Einstein relation.[7] For Stokes radiation in the 1550-nm range, g_R = 70 cm/GW, a value that is 10^4 times larger than the Raman gain in silica fiber! The third-order nonlinear susceptibility that describes stimulated emission can be calculated in terms of the Raman tensor as[9]

$$\chi^{(3)}_{jkmn} \sim \sum_{i=1,2,3} (R_i)_{jk} (R_i)_{mn}\,. \qquad (4)$$

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The nonlinear polarization then has the form

$$P_i^{(\mathrm{NL})}(\omega_s) = \epsilon_0 \chi_{ijmn}^{(3)} E_j(\omega_p) E_m(-\omega_p) E_n(\omega_s)\,. \tag{5}$$

In a single-mode SOI waveguide the two propagating modes are a quasi-TE and a quasi-TM mode. We assume that the pump power (at 1434 nm) is coupled to the quasi-TE mode, and we consider the Stokes wave to be a linear combination of the lowest-order TE and TM modes (propagation along $y = [1\bar{1}0]$):

$$\vec{\mathbf{E}}(\omega_s) = \sum_{\mu=1,2} A_{s,\mu}(y)\vec{\boldsymbol{\epsilon}}_\mu(\omega_s, x, z)\exp(j\beta_\mu y)\,,$$

$$A_{s,1}(0) = \sqrt{aP_s}\,, \qquad A_{s,2}(0) = \sqrt{(1-a)P_s}\,,$$

$$\vec{\mathbf{E}}(\omega_p) = A_{p,1}(y)\vec{\boldsymbol{\epsilon}}_1(\omega_p, x, z)\exp[j\beta_1(\lambda_p)y]\,,$$

$$A_{p,1}(0) = \sqrt{P_p}\,, \tag{6}$$

where $\mu = 1 \equiv$ TE and $\mu = 2 \equiv$ TM and the modes $\vec{\boldsymbol{\epsilon}}_\mu(\omega, x, z)$ are normalized to unit power, with a representing the ratio of power in the TE Stokes mode. The evolution of the amplitudes $A_{s,\mu}$ of the Stokes wave along the waveguide is derived from coupled-mode theory.[10] Assuming no pump depletion,

$$\frac{\mathrm{d}A_{s,1}}{\mathrm{d}y} = i\kappa_{11}P_pA_{s,1} + i\kappa_{12}P_pA_{s,2}\exp[i(\beta_2 - \beta_1)y]\,,$$

$$\frac{\mathrm{d}A_{s,2}}{\mathrm{d}y} = i\kappa_{22}P_pA_{s,2} + i\kappa_{21}P_pA_{s,1}\exp[-i(\beta_2 - \beta_1)y]\,, \tag{7}$$

where P_p denotes the coupled pump power and the coupling coefficients $[\mathrm{m}^{-1}\,\mathrm{W}^{-1}]$ are

$$\kappa_{\mu\mu'} = -j \sum_{i,j,m,n=1,2,3} \frac{4}{Z^2}\frac{g_R}{2}\xi_{ijmn} \iint [\epsilon_\mu{}^i(\omega_s)]^* \epsilon_1{}^j(\omega_p) \times [\epsilon_1{}^m(\omega_p)]^* \epsilon_{\mu'}{}^n(\omega_s)\mathrm{d}x\mathrm{d}z\,, \tag{8}$$

where $Z = Z_0/n_{\mathrm{Si}}$ is the impedance in the waveguide, $\xi_{ijmn} = \chi_{ijmn}^{(3)}/\chi_{1111}^{(3)}$, and i and j are the label vector components of the pump and the Stokes fields, respectively. $\epsilon_\mu{}^i(\omega)$ is the mode profile for the ith component of the μth mode. The susceptibility is written in terms of gain coefficient $\chi_{1111}^{(3)} = 2ncg_R/(\omega_s Z)$. In the case of weak coupling between the modes ($k_{12} \ll k_{11}, k_{22}$), which is the case here, the gain, G, for the TE (TM) polarization will be roughly given by $G_{\mathrm{TE\,(TM)}} \approx 4.34 \times 2|\kappa_{11\,(22)}|P_p$ [dB/cm].

For the spontaneous effect the Stokes amplitudes (one induced polarization per phonon branch λ) for each Stokes mode μ in the waveguide are

$$\frac{\mathrm{d}A_{s,\mu,\lambda}}{\mathrm{d}y} = i\kappa_{\mu,\lambda}A_p\,, \tag{9}$$

with the coupling coefficients equal to

$$\kappa_{\mu,\lambda} = \omega_s\epsilon_0\chi_R \iint [\vec{\boldsymbol{\epsilon}}_\mu(\omega_s)]^* \cdot \vec{\mathbf{R}}_\lambda \cdot \vec{\boldsymbol{\epsilon}}_1(\omega_p)\mathrm{d}x\mathrm{d}z\,. \tag{10}$$

For a waveguide of length l, the scattering efficiency at the Stokes mode μ $[\mathrm{m}^{-1}]$ is

$$S_\mu = \frac{1}{l}\frac{|A_{s,\mu}|^2}{|A_p|^2} = l(|\kappa_{\mu,1}|^2 + |\kappa_{\mu,2}|^2 + |\kappa_{\mu,3}|^2)\,. \tag{11}$$

From Eqs. (3), (10), and (11) we get

$$S_\mu = S_0\frac{32\pi^2}{k_0{}^4}\omega^2\epsilon_0{}^2 \times \frac{\sum_{\lambda=1,2,3}|\iint[\vec{\boldsymbol{\epsilon}}_\mu(\omega_s)]^*\cdot\vec{\mathbf{R}}_\lambda\cdot\vec{\boldsymbol{\epsilon}}_1(\omega_p)\mathrm{d}x\mathrm{d}y|^2}{A_{\mathrm{eff}}}\,. \tag{12}$$

For Eq. (12), we evaluated the volume in Eq. (3) as $A_{\mathrm{eff}}l$, with A_{eff} being the effective area of the TE incident mode,

$$A_{\mathrm{eff}} = \frac{[\iint|\vec{\boldsymbol{\epsilon}}_1(\omega_p)|^2\mathrm{d}x\mathrm{d}z]^2}{\iint|\vec{\boldsymbol{\epsilon}}_1(\omega_p)|^4\mathrm{d}x\mathrm{d}z}\,. \tag{13}$$

We now provide results for a variety of waveguide dimensions. The waveguide geometry is depicted in Fig. 1. We consider rib waveguides of rib width w, slab height h, and rib height H, with $w/h = 2/1.4$ and $w/H = 2/2.15$. Rib waveguides with these ratios are shown to be single mode for waveguide widths down to $w \approx 0.5\ \mu\mathrm{m}$.[11] The case that we examine is for waveguides oriented along $[0\bar{1}1]$ in a [100] wafer. The mode profiles of the waveguide are obtained by a finite-difference time domain method[12] and are used in the numerical evaluation of the coupling coefficients.

The calculated coupling coefficients and scattering efficiencies are shown in Table 1. Coefficients κ_{11} and κ_{22} and efficiencies S_{TE} and S_{TM} are close to what is expected from the theory in the bulk.

The fact that the spontaneous efficiencies for the TE mode are larger than the TM mode, whereas the converse is true for gain coefficients, is not entirely surprising. This is in contradiction to what is predicted by the Einstein relation. The validity of the Einstein relation between efficiency and gain is limited to fields with spatially homogeneous and isotropic energy density.[13] As the waveguide dimensions are reduced, the

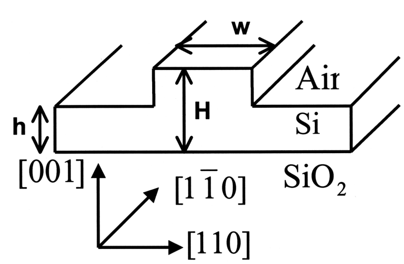

Fig. 1. Rib-waveguide geometry.

Table 1. Coupling Coefficients and Spontaneous Efficiency for a Rib Waveguide with $w/h = 2/1.4$ and $w/H = 2/2.15$

w (μm)	$A_R{}^{\text{eff}}$ TE (μm^2)	$A_R{}^{\text{eff}}$ TM (μm^2)	κ_{11} TE (cm W)$^{-1}$	κ_{22} TM (cm W)$^{-1}$	κ_{12} ($\times 10^{-3}$) (cm W)$^{-1}$	S_{TE} (cm^{-1})	S_{TM} (cm^{-1})
4	16.2	15.3	$-j0.215$	$-j0.23$	$-0.32 - j0.58$	19.6×10^{-9}	19.2×10^{-9}
3	8.9	8.25	$-j0.39$	$-j0.42$	$-2.5 + j1.36$	35.8×10^{-9}	35.2×10^{-9}
2	4.3	3.9	$-j0.82$	$-j0.89$	$-11.9 + j13.3$	69.2×10^{-9}	67.0×10^{-9}
1.2	1.6	1.4	$-j2.16$	$-j2.45$	$-284 + j89.7$	192×10^{-9}	183×10^{-9}
0.8	1.1	0.9	$-j3.1$	$-j3.88$	$212 - j69.5$	271.2×10^{-9}	222.0×10^{-9}
0.4	0.42	0.36	$-j8.28$	$-j9.7$	$-58.9 - j18$	787.0×10^{-9}	540.0×10^{-9}

deviation from the homogeneous energy density becomes more pronounced, and so does the effect mentioned above.

The spontaneous efficiency and the gain coefficients are equal for the [001] and [110] Stokes polarizations in bulk scattering with the specified geometry. We introduce effective area $A_{\text{eff}}{}^R$ (not to be confused with the notation A_{eff} used above for the effective area of the pump TE mode), defined such that $2\kappa P_p = g_R I_p = g_R P_p / A_{\text{eff}}{}^R$ ($\kappa = |\kappa_{11}|$ for TE and $|\kappa_{22}|$ for TM). This area is listed in Table 1 and can be used to calculate the Raman gain from the bulk gain value, g_R. The effective area is proportional to the rib area down to $w = 0.4$ μm, and thus the gain increases with a decrease of the waveguide dimensions.

As one can see from the calculated coefficients, κ_{12} is 2–3 orders of magnitude smaller than κ_{11}, and the two Stokes modes propagate independently. Since the TE and TM polarizations are decoupled in principle, two signals can be amplified by polarization multiplexing. The two signals can be introduced in a SOI waveguide in orthogonal polarizations (TE and TM) and can both be amplified without interference. Moreover, the gain coefficients are almost equal for the TE and TM polarizations, and thus the gain is highly polarization insensitive.

In summary, we have presented what are believed to be the first coupled-mode calculations of Raman gain in silicon waveguides. The results indicate that there is negligible modal coupling for the range of waveguide dimensions considered. With the proper definition of a mode area, bulk gain values can be used to calculate the Raman gain for a waveguide.

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